A Multi-Dimensional System of Fracture Abundance Measures

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Outline of Presentation

- Related systems and concepts
- Basic definitions
- Terms currently used to describe amount of fracturing
- Proposed terminology
- Assumptions
- Sampling problems
- Fracture abundance measures - density, intensity & porosity
- Measuring density
- Measuring intensity
- Measuring porosity
- Converting among abundance measures
- Conclusions

Related Systems and Concepts

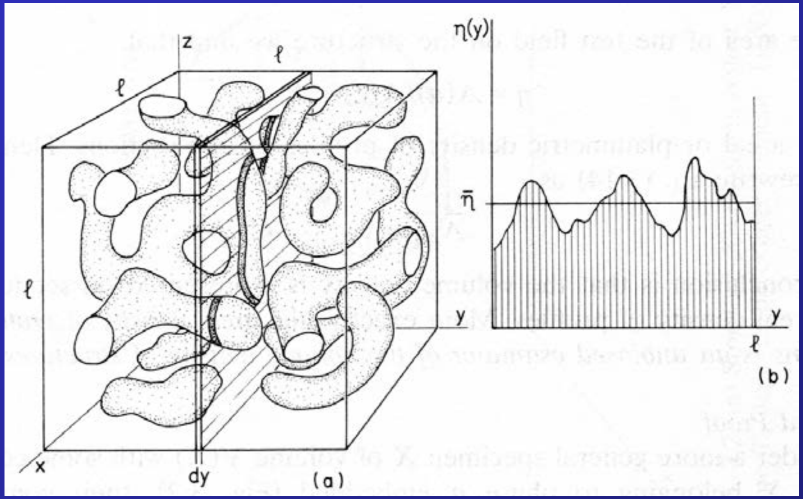
Stereology, Stochastic geometry

IUR sample

$V_V = A_A$ (Delesse, 1848)

$V_V = A_A = L_L$ (Rosiwal, 1898)

$V_V = A_A = L_L = P_P$ (Thomson, 1930)



Geologists and rock engineers deal primarily with oriented structures and oriented sampling domains

Fabric tensor, crack tensor, anisotropic fracture networks

Krumbein (J. Geol. 1939); Scheidegger (USGS 1965); Kiraly (Geol. Rundschau 1969); Ruhland (Geol. Soc. Bull. 1973); Van Golf-Racht (Fund. of Fractured Reservoir Engng. 1982) Kanatani (Int. J Engng. Sc. 1984); Oda (Geotechnique 1985); Long & Witherspoon (JGR 1985); Dershowitz & Einstein (RMRE 1988); Dershowitz & Herda (USRM Symp 1992) Mauldon (IJRMMS 1994) Marrett, (JSG 1996)

Basic definitions

Fracture - generic term for rock discontinuities of all types. The term embraces joints, faults, veins, shear zones, foliation planes, bedding surfaces and other lithological boundaries, and contacts.

Fracture area (A) - the average area of the two faces of a fracture.

Fracture area per unit volume (A/V) - mean summed fracture area enclosed within a specified volume, divided by that volume

Fracture aperture (e) - average mechanical aperture of a fracture.

Fracture trace length (L) - The length of a fracture trace formed as the intersection between a fracture and a 2-d sampling surface.

Fracture trace length per unit area (L/A) - mean summed trace length enclosed within a specified area, divided by that area

Sampling domain - the region on which fracturing is to be measured or estimated. May be 3-d (e.g., the rock to be excavated for a tunnel), 2-d (e.g., rock pavements, pre-split road crops, tunnel walls) or 1-d (e.g., scanlines or small-diameter boreholes).

Terms currently used to describe amount of fracturing (1)

Fracture spacing

Fracture density

Fracture intensity

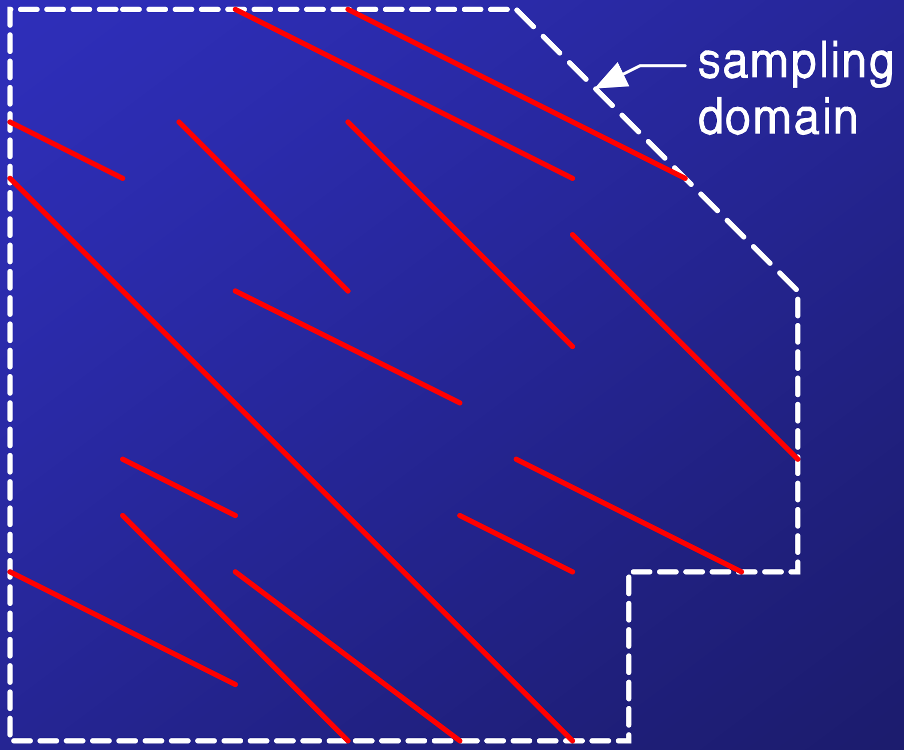
Fracture porosity

Fracture frequency

Fracture persistence

Dimensionless fracture density

Degree of development



Terms currently used to describe amount of fracturing (2)

Fracture spacing	*ambiguous; various interpretations for non-parallel or non-infinite fractures*
Fracture density	*various definitions, no consensus, often used in a qualitative sense*
Fracture intensity	*various definitions, no consensus, often used in a qualitative sense*
Fracture porosity	*well defined quantity; can be applied in 1, 2 or 3-dimensions*
Fracture frequency	*OK for a specified sampling line. Direction-dependency must be noted explicitly*
Fracture persistence	*OK for intermittent or partially healed joints; otherwise ambiguous*
Dimensionless fracture density	*used in fracture mechanics; not useful for geologic fractures*
Degree of development	*OK, but qualitative*

Proposed Terminology

Fracture abundance:

umbrella term for amount of fracturing:

Scale-independent

Fracture abundance measures:

Density, Intensity, Porosity

defined in 1, 2 and 3-dimensions

Assumptions

- Fractures may be systematic or non-systematic, planar or non-planar, convex or non-convex.

- Fractures may possess cross-cutting, en echelon, termination, or any type of relationship with other fractures or structures.

- No spatial distributions are assumed.

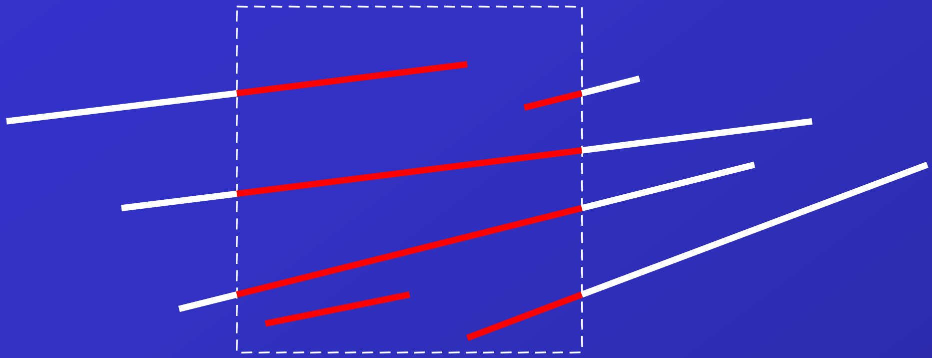
- No particular orientation distributions are assumed, although certain special cases are examined.

We do assume that:

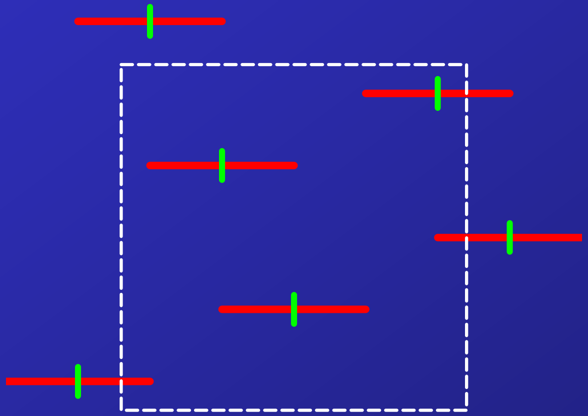
- Fracture distributions and locations are **independent** of the sampling domain.

- Fracture aperture is significantly smaller than diameter.

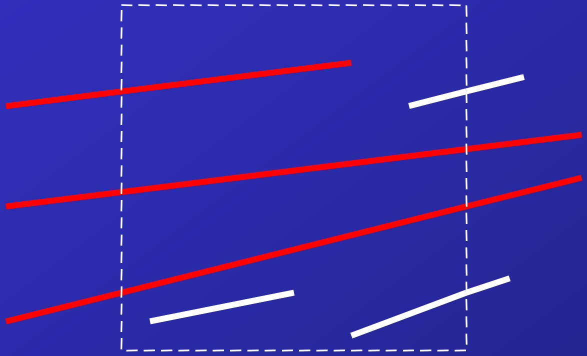
Sampling Problems



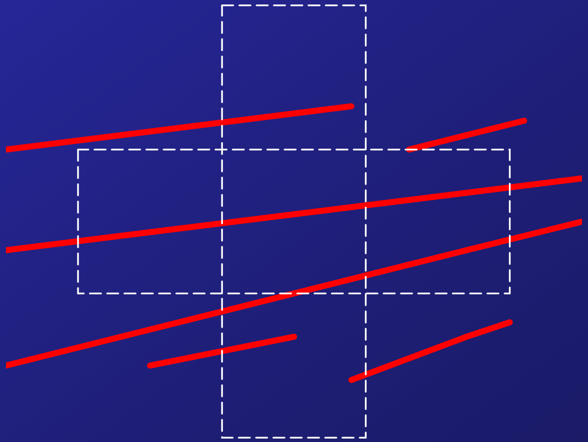
Censoring ❄



Length-bias ❄



Truncation



Orientation ❄

Einstein and Baecher, RMRE 1983; Odling, JSG 1997; Mauldon RMRE 1998

Fracture Abundance Measures - **Density**

Fracture Density

$^{*}P_{10}$ (number of fractures / length of scanline) $[L^{-1}]$

$^{*}P_{20}$ (number of fractures / area of exposure) $[L^{-2}]$

P_{30} (number of fractures / volume of rock mass) $[L^{-3}]$

* direction-dependent

Fracture Abundance Measures - **Intensity**

Fracture Intensity

*P_{10}(number of fractures / length of scanline) $[L^{-1}]$

*P_{21} (length of fracture traces / area of exposure) $[L^{-1}]$

P_{32} (area of fractures / volume of rock mass) $[L^{-1}]$

* direction-dependent

Fracture Abundance Measures - **Porosity**

Fracture Porosity

$^*P_{11}$ (thickness of fractures / length of scanline) [-]

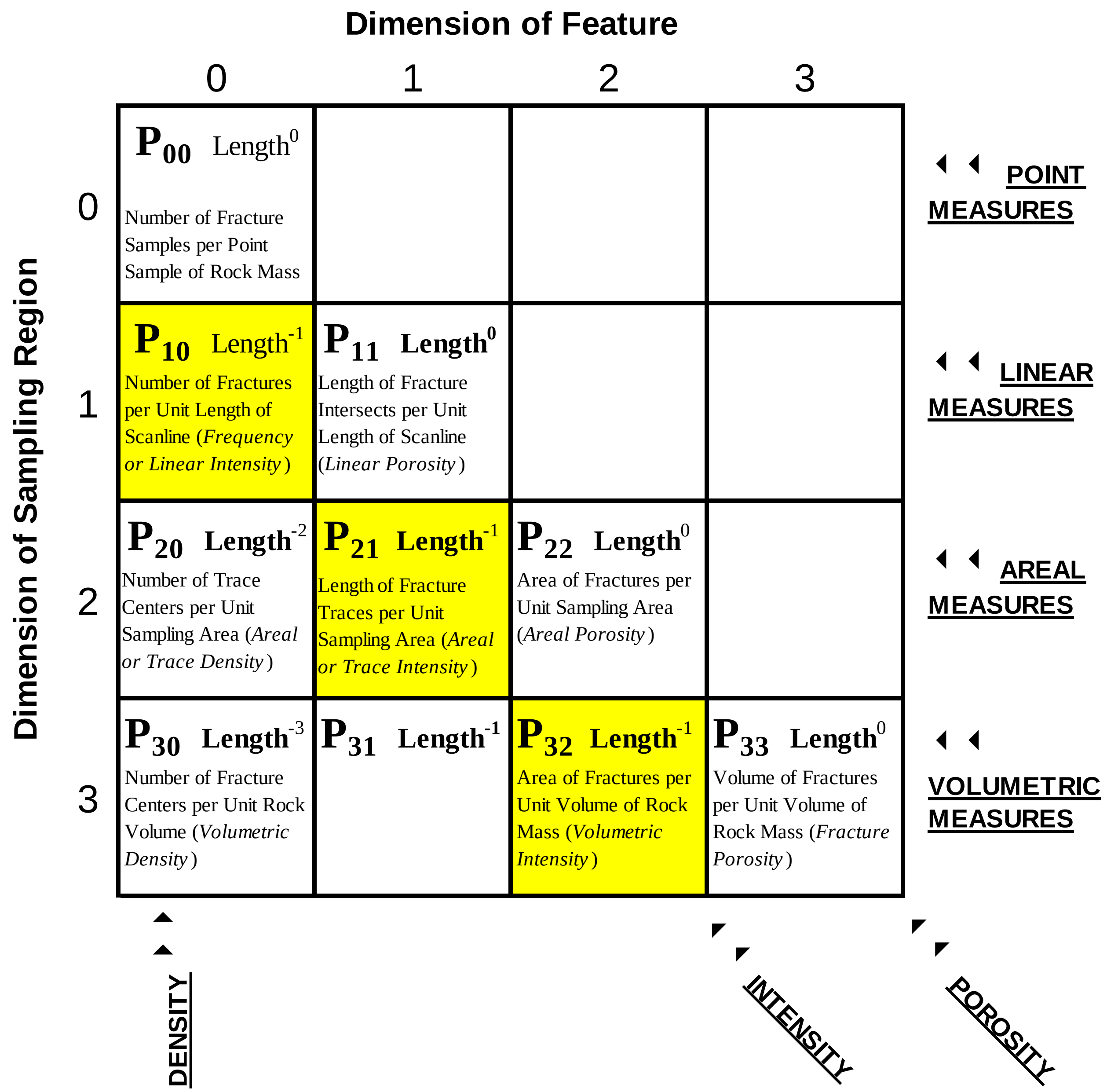
$^*P_{22}$ (area of fracture traces / area of exposure) [-]

P_{33} (volume of fractures / volume of rock mass) [-]

* direction-dependent

Fracture Abundance measures

Dershowitz & Herda, USRM Symp. 1992;
Mauldon & Dershowitz (in preparation, IJRMMS)

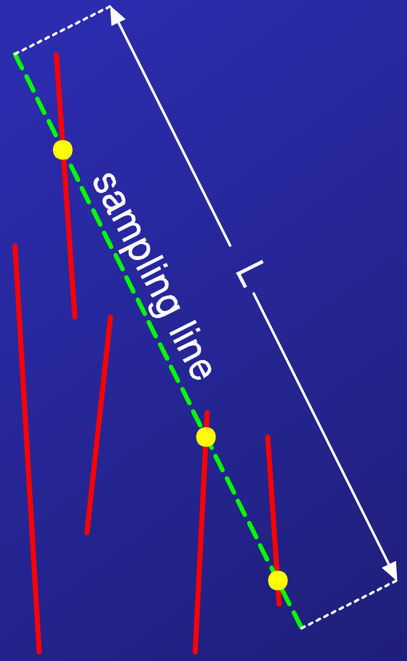


Dimension of Feature

Dimension of Sampling Region

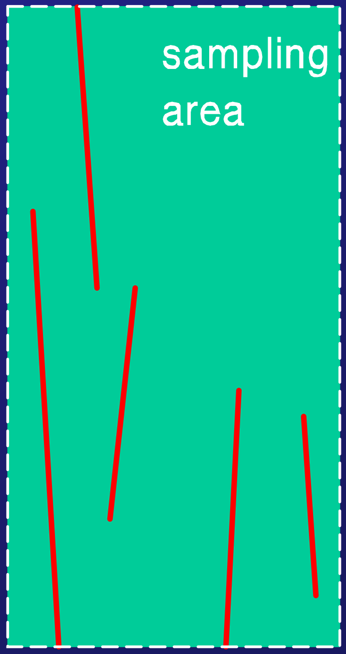
	0	1	2	3	
0	**P_{00}** Length^{0} Number of Fracture Samples per Point Sample of Rock Mass				**POINT MEASURES**
1	**P_{10}** Length^{-1} Number of Fractures per Unit Length of Scanline (*Frequency or Linear Intensity*)	**P_{11}** **Length^{0}** Length of Fracture Intersects per Unit Length of Scanline (*Linear Porosity*)			**LINEAR MEASURES**
2	**P_{20}** **Length^{-2}** Number of Trace Centers per Unit Sampling Area (*Areal or Trace Density*)	**P_{21}** **Length^{-1}** Length of Fracture Traces per Unit Sampling Area (*Areal or Trace Intensity*)	**P_{22}** **Length^{0}** Area of Fractures per Unit Sampling Area (*Areal Porosity*)		**AREAL MEASURES**
3	**P_{30}** **Length^{-3}** Number of Fracture Centers per Unit Rock Volume (*Volumetric Density*)	**P_{31}** **Length^{-1}**	**P_{32}** **Length^{-1}** Area of Fractures per Unit Volume of Rock Mass (*Volumetric Intensity*)	**P_{33}** **Length^{0}** Volume of Fractures per Unit Volume of Rock Mass (*Fracture Porosity*)	**VOLUMETRIC MEASURES**

Measuring Density (1)



$P_{10}(\underline{\theta})$ = number of fractures / length of scanline

Direct field measurements N/L along a straight line give unbiased estimates of $P_{10}(\underline{\theta})$

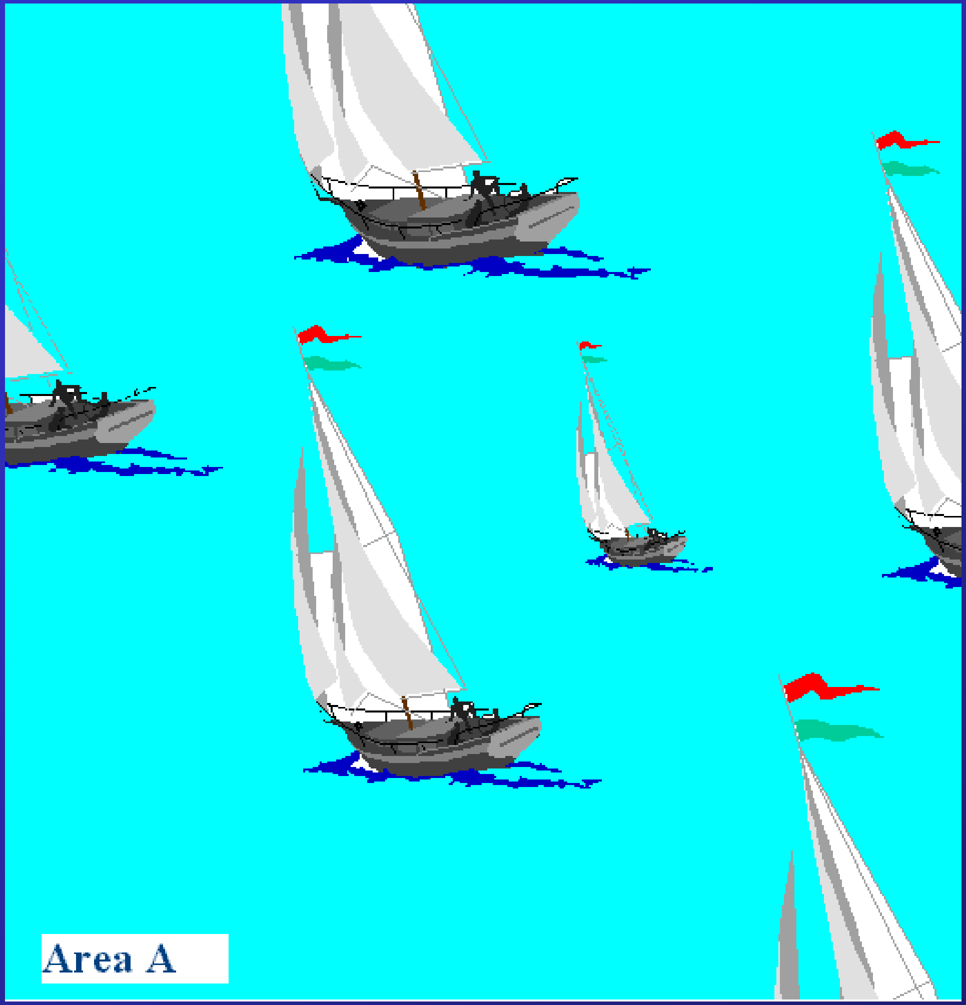


$P_{20}(\underline{\theta})$ = number of fractures / area of exposure

Direct field measurements N/A on a planar surface give unbiased estimates of $P_{20}(\underline{\theta})$ *as long as the number of fractures is determined correctly.*

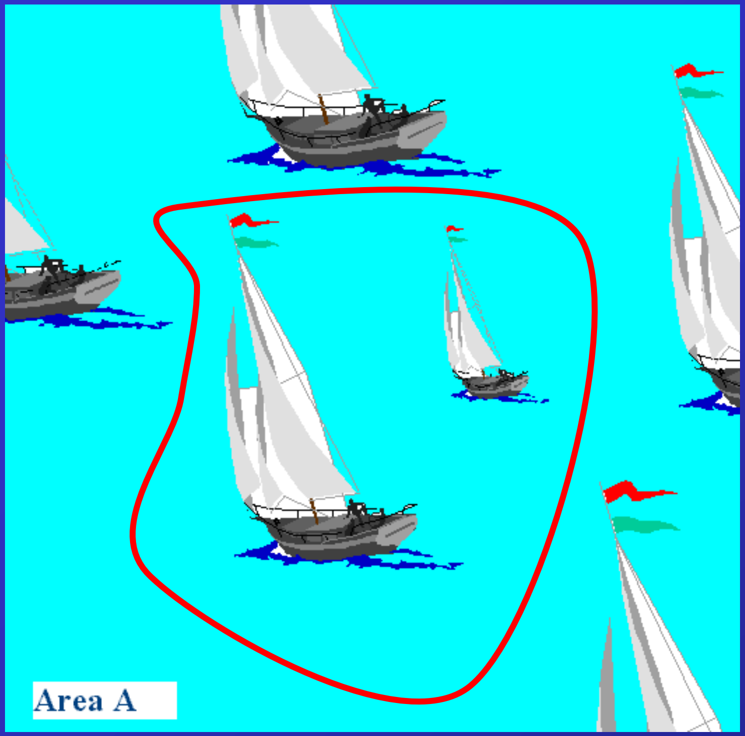
Question: What is the <u>number</u> of fractures for measurements of P_{20}? (and P_{30}?)

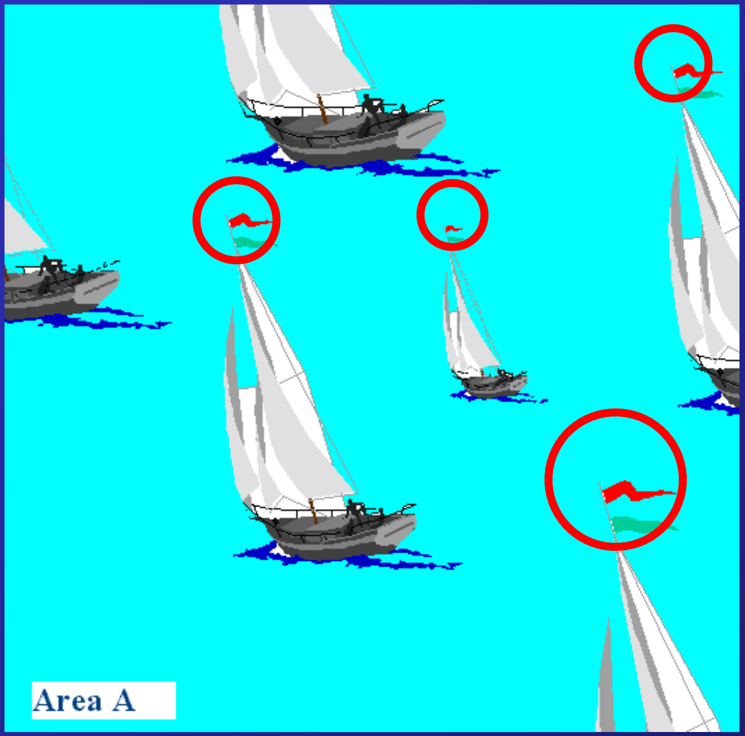
Measuring Density (2)



How many sailboats per unit area?

Measuring Density (3)



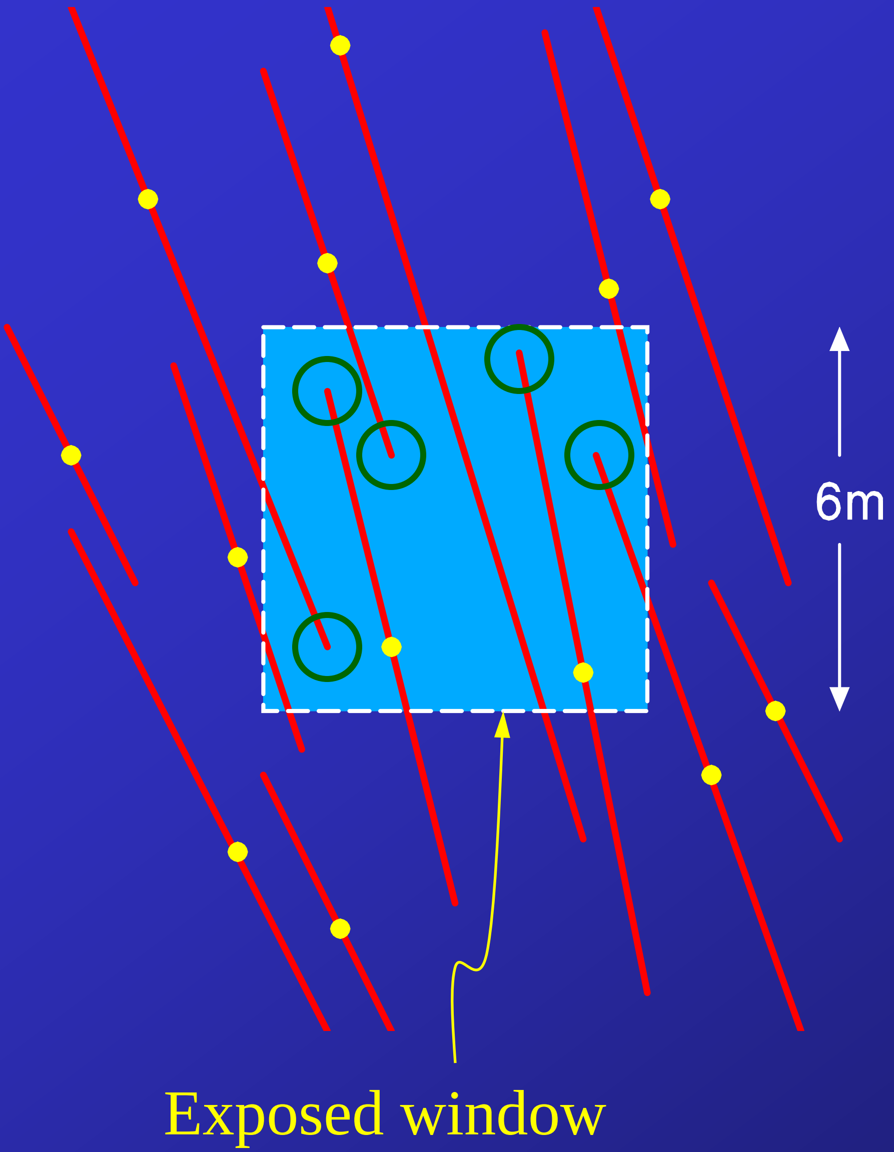


Not recommended: Identify subdomain containing uncensored sailboats. This violates assumed independence of structure and sampling domain; results are invalid.

Recommended: Identify and count unique associated points within sampling domain. This gives a scale-independent , unbiased estimate.

Sailboat density P_{20}= # red flags/A = half total # of flags/A = 4/A.

Measuring Density (4)



Note two-to-one association between trace ends and centers.

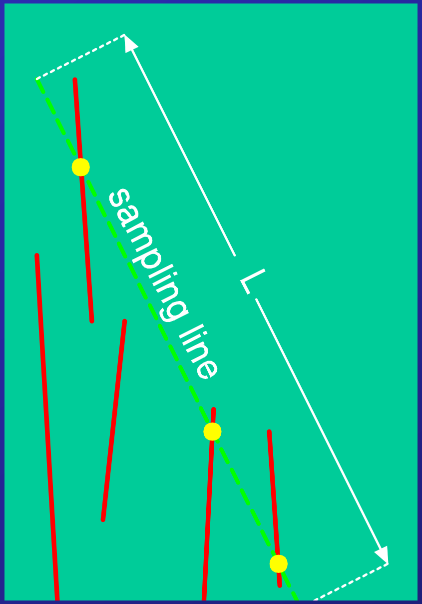
An unbiased estimate of trace density is therefore given by ½ number of trace ends divided by window area. For this example,

$$\hat{P}_{20} = \frac{5/2}{36m^2} = 0.07m^{-2}$$

Estimator is scale-independent

Mauldon, RMRE 1992

Measuring Intensity (1)



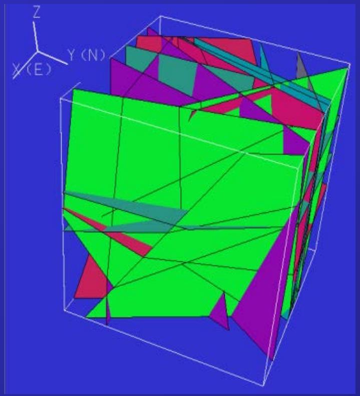
P_{10} = number of fractures / length of scanline

SCANLINES: Direct field measurements N/L along a straight line give unbiased estimates of $P_{10}(\underline{\theta})$

P_{21} = length of fracture traces / area of exposure

AREAS: Direct field measurements L'/A on a planar exposure give unbiased estimates of $P_{21}(\underline{\theta})$

P_{32} = area of fractures / volume of rock mass

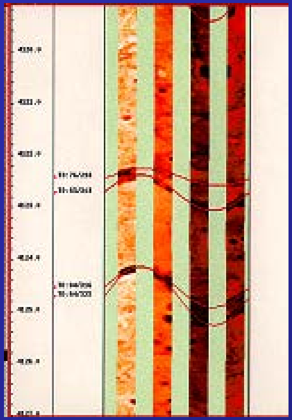


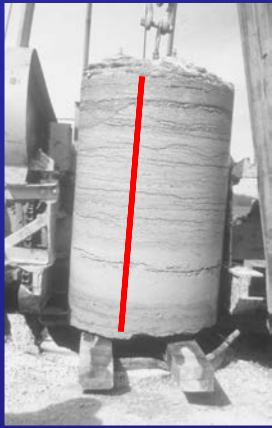
VOLUMES: Direct field measurements A'/V give unbiased estimates of $P_{21}(\underline{\theta})$ *--- In principle! - impossible to obtain directly)*

P_{32} *is the "Holy Grail" of fracture abundance measures*

Measuring Intensity (2)

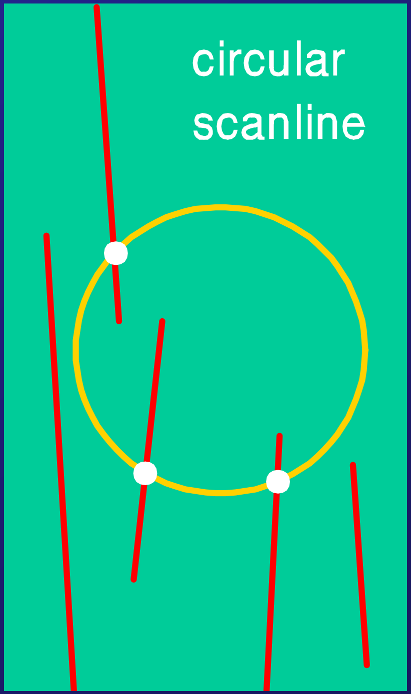
P_{10} = number of fractures/length of borehole





BOREHOLES/ ROCK CORE: Direct field measurements N/L along a borehole give unbiased estimates of $P_{10}(\theta)$

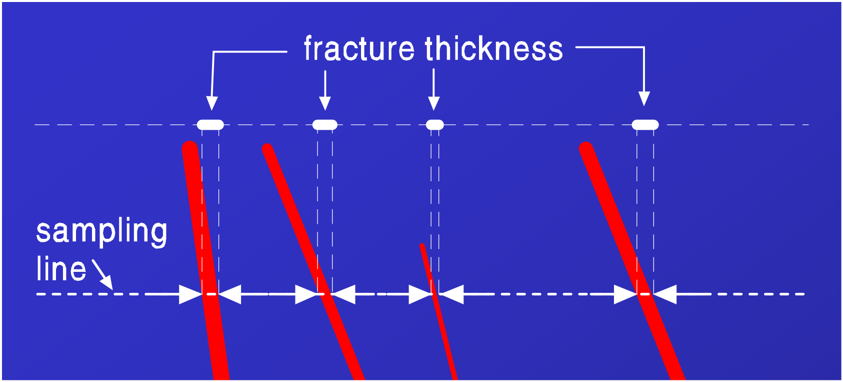
P_{21} = length of fracture traces / area of exposure



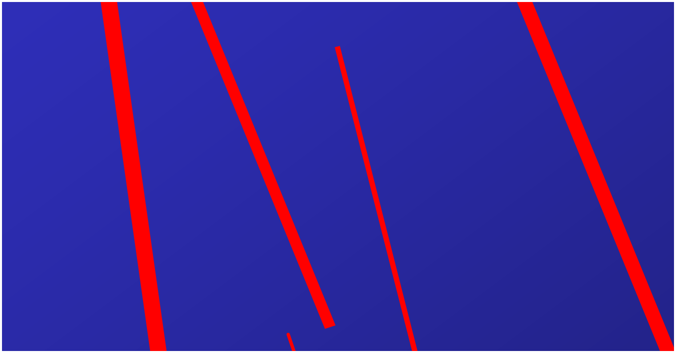
CIRCULAR SCANLINES: Direct field measurements of n/4r on a circular scanline , where n is the number of trace intersections on the circle and r is the radius of the circle, give unbiased estimates of P_{21} *(*θ*) on a planar exposure.*

Mauldon, Dunne & Rohrbaugh, JSG, in press

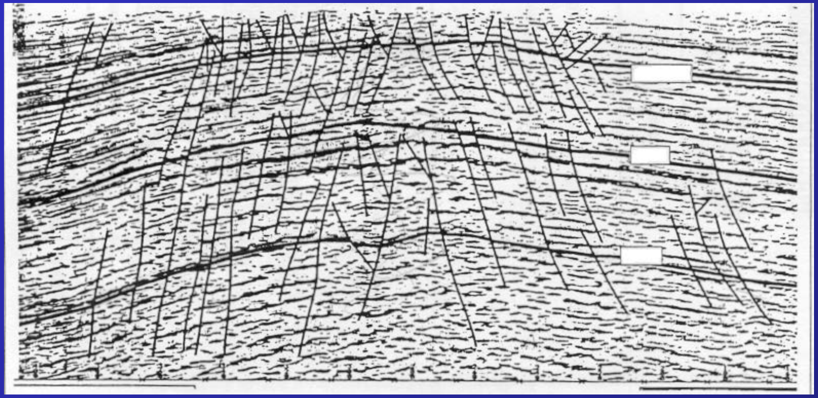
Measuring Porosity



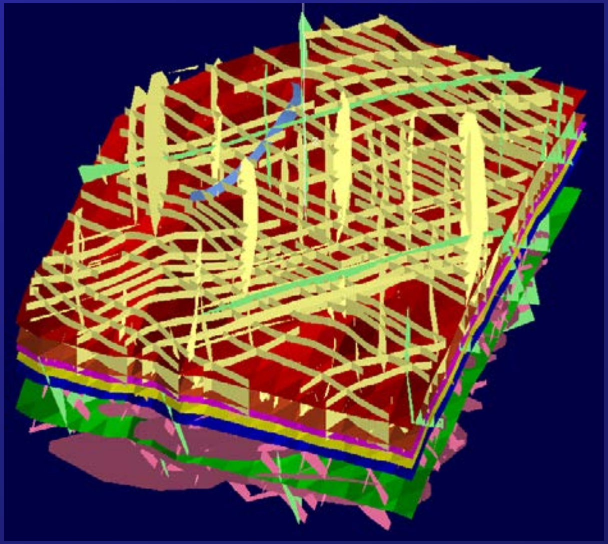
Linear porosity



Areal porosity



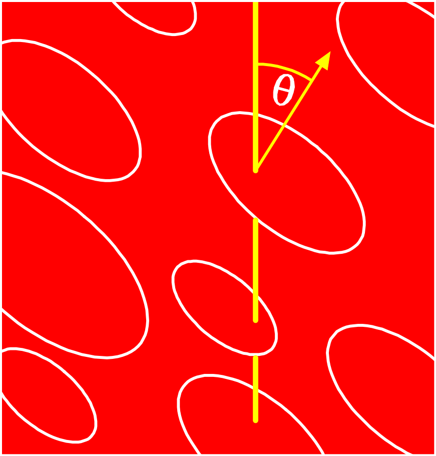
Faults mapped directly from VSP vertical seismic profiling



Fractured oil reservoir - one-kilometer scale DFN model

Converting among Abundance Measures

--- A few examples ---



Parallel fractures

$P_{32} = \csc(\theta)P_{10}(\theta)$ (R. Terzaghi, 1965)

Uniform distribution of fracture orientations

$P_{32} = 2\ P_{10}$ from 1-d Measures

$P_{32} = (4/\pi)\ P_{21}$ from 2-d Measures

Specified distribution of fracture orientation

$P_{32} = C_{31}\ P_{10}$ from 1-D Measures

$P_{32} = C_{32}\ P_{21}$ from 2-D Measures

Conversion factors

Some conversion factors are known analytically

Conversion factors can be developed by simulation

Dershowitz & Herda, USRM Symp. 1992;
Mauldon & Dershowitz (in preparation, IJRMMS)

A Multi-Dimensional System of Fracture Abundance Measures

Conclusions

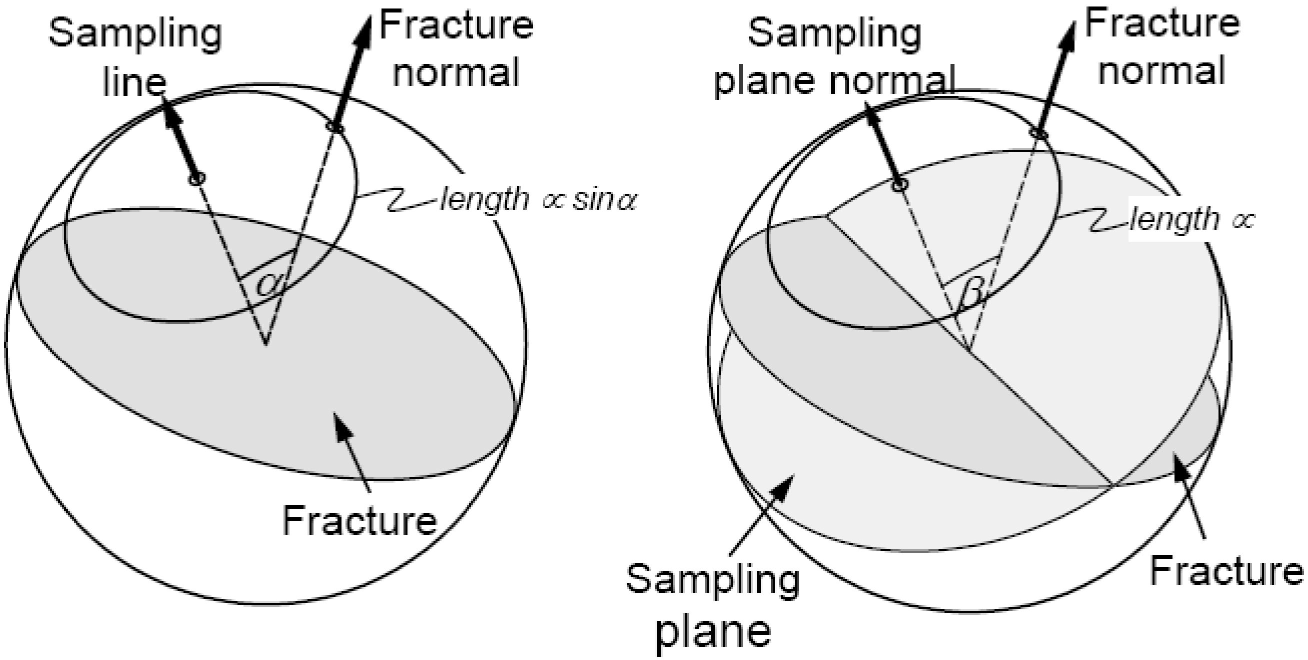
- The amount of fractures present in a rock mass can be described by fracture abundance measures: density, intensity and porosity.
- The abundance measures are scale-independent and are applicable in 1, 2 or 3-dimensions
- 1-d and 2-d abundance measures are orientation-dependent
- Estimates of these measures can be obtained from scanlines, scan-circles, borehole images, windows and tunnels.
- Together, the fracture abundance measures form a unified, self-consistent system

Amroth, South Wales, UK

Thank You

Wang(2006) Intensity Conversion:
C31 and C32

Fracture Orientation Relative to Sampling Lines (Boreholes) and Planes (Tracemaps)

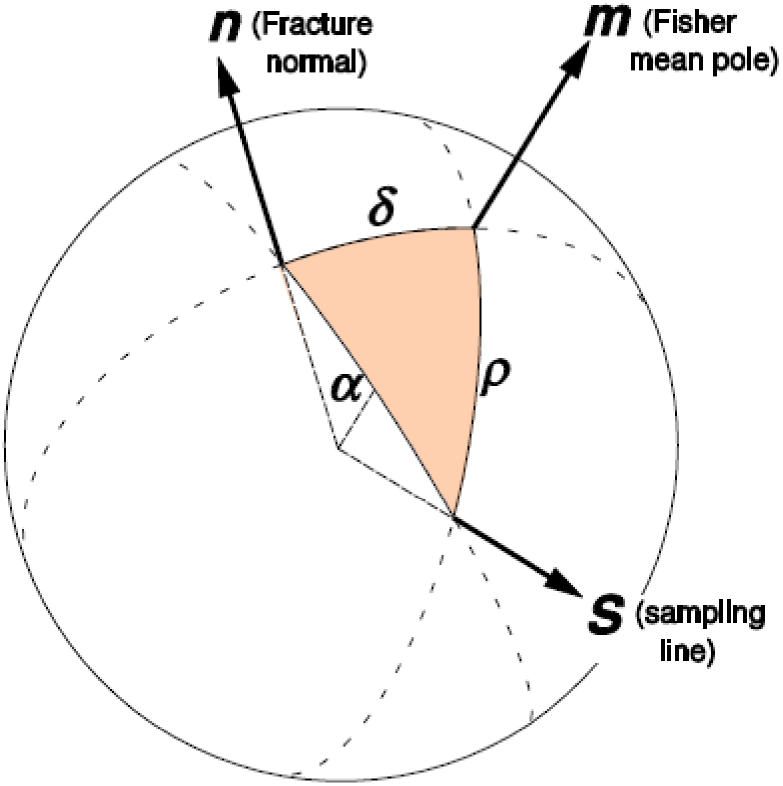


(a)

(b)

After Wang (2006)

Angle ρ between fracture set mean pole and sampling line (borehole)



After Wang (2006)

Calculation of C_{31} Conversion between P_{10} and P_{32} for Fisher Distributed Fractures

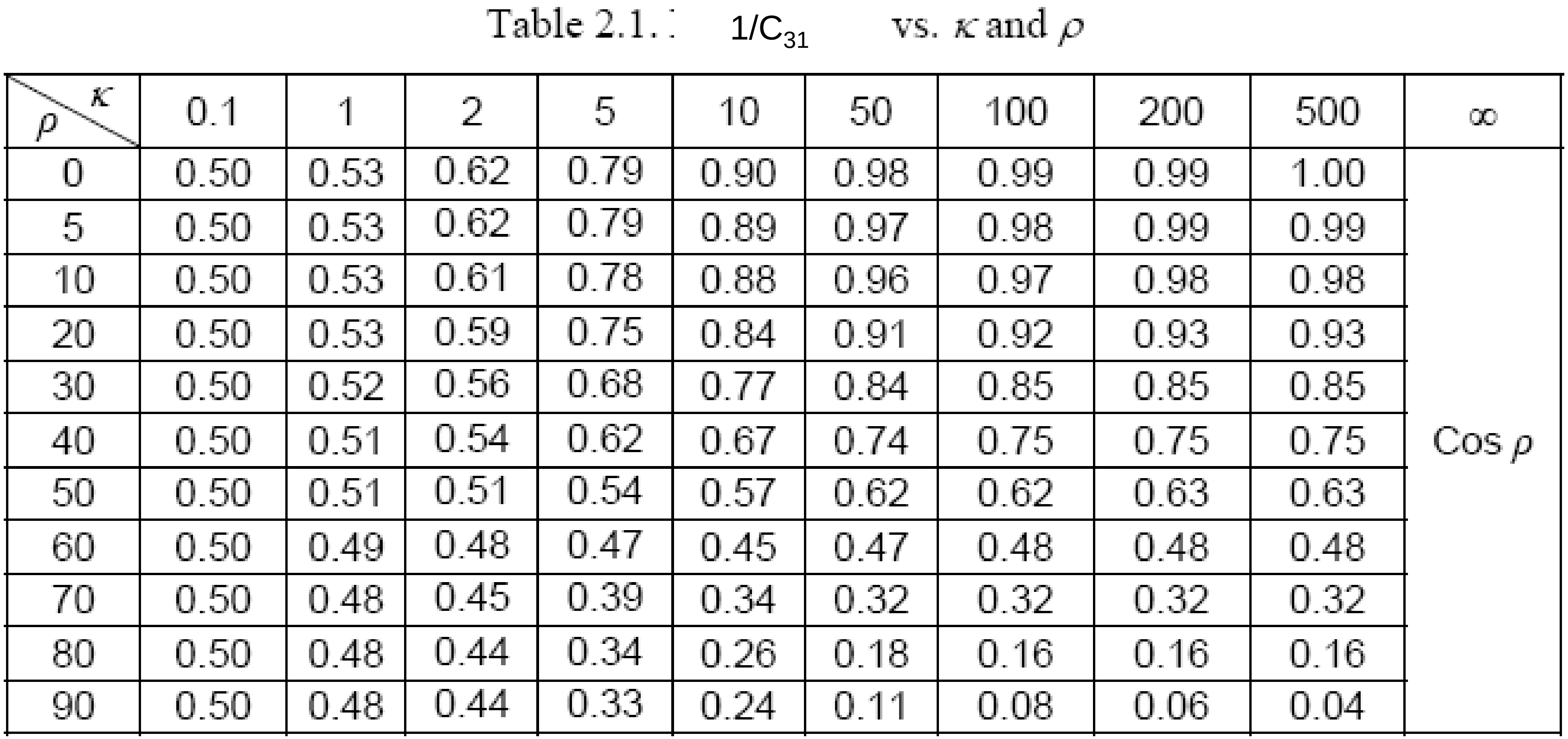
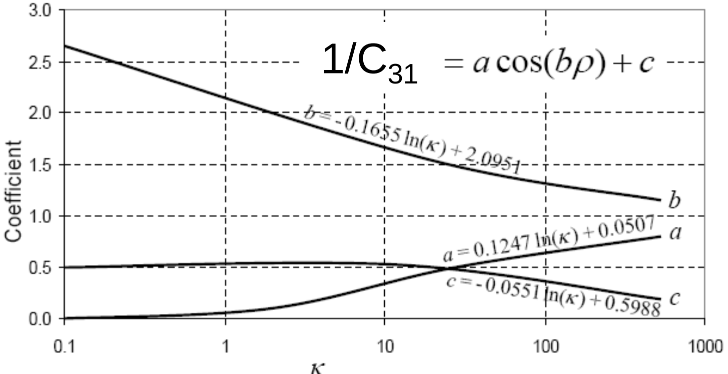


Table 2.1. 1/C_{31} vs. κ and ρ

ρ \ κ	0.1	1	2	5	10	50	100	200	500	∞
0	0.50	0.53	0.62	0.79	0.90	0.98	0.99	0.99	1.00	Cos ρ
5	0.50	0.53	0.62	0.79	0.89	0.97	0.98	0.99	0.99	
10	0.50	0.53	0.61	0.78	0.88	0.96	0.97	0.98	0.98	
20	0.50	0.53	0.59	0.75	0.84	0.91	0.92	0.93	0.93	
30	0.50	0.52	0.56	0.68	0.77	0.84	0.85	0.85	0.85	
40	0.50	0.51	0.54	0.62	0.67	0.74	0.75	0.75	0.75	
50	0.50	0.51	0.51	0.54	0.57	0.62	0.62	0.63	0.63	
60	0.50	0.49	0.48	0.47	0.45	0.47	0.48	0.48	0.48	
70	0.50	0.48	0.45	0.39	0.34	0.32	0.32	0.32	0.32	
80	0.50	0.48	0.44	0.34	0.26	0.18	0.16	0.16	0.16	
90	0.50	0.48	0.44	0.33	0.24	0.11	0.08	0.06	0.04	

$$P_{32} = C_{31} P_{10}$$

After Wang (2006)



Calculation of C_{32} Conversion between P_{21} and P_{32} for Fisher Distributed Fractures

Table 2.2.1 $1/C_{32}$ with different values of κ and ρ.

κ / ρ	0.1	1	2	5	10	50	100	200	500	∞
0	0.79	0.77	0.70	0.53	0.39	0.19	0.14	0.11	0.07	Sin ρ
5	0.79	0.77	0.70	0.54	0.40	0.21	0.17	0.14	0.12	
10	0.79	0.77	0.70	0.54	0.43	0.25	0.22	0.20	0.20	
20	0.79	0.77	0.72	0.58	0.49	0.38	0.37	0.36	0.36	
30	0.79	0.78	0.74	0.64	0.58	0.53	0.52	0.52	0.52	
40	0.79	0.78	0.76	0.71	0.68	0.66	0.66	0.66	0.66	
50	0.79	0.78	0.78	0.77	0.77	0.77	0.77	0.78	0.78	
60	0.79	0.79	0.79	0.82	0.85	0.87	0.87	0.87	0.87	
70	0.79	0.80	0.82	0.87	0.90	0.94	0.94	0.94	0.94	
80	0.79	0.80	0.83	0.90	0.94	0.98	0.98	0.99	0.99	
90	0.79	0.80	0.83	0.91	0.95	0.99	0.99	1.00	1.00	

$$P_{32} = C_{32} P_{21}$$

After Wang (2006)