A Multi-Dimensional System of Fracture Abundance Measures

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Outline of Presentation

- Related systems and concepts
- Basic definitions
- Terms currently used to describe amount of fracturing
- Proposed terminology
- Assumptions
- Sampling problems
- Fracture abundance measures - density, intensity & porosity
- Measuring density
- Measuring intensity
- Measuring porosity
- Converting among abundance measures
- Conclusions
Related Systems and Concepts

Stereology, Stochastic geometry

IUR sample

$V_V = A_A$ (Delesse, 1848)

$V_V = A_A = L_L$ (Rosial, 1898)

$V_V = A_A = L_L = P_P$ (Thomson, 1930)

Geologists and rock engineers deal primarily with oriented structures and oriented sampling domains

Fabric tensor, crack tensor, anisotropic fracture networks

**Basic definitions**

**Fracture** - generic term for rock discontinuities of all types. The term embraces joints, faults, veins, shear zones, foliation planes, bedding surfaces and other lithological boundaries, and contacts.

**Fracture area (A)** - the average area of the two faces of a fracture.

**Fracture area per unit volume (A/V)** - mean summed fracture area enclosed within a specified volume, divided by that volume.

**Fracture aperture (e)** - average mechanical aperture of a fracture.

**Fracture trace length (L)** - The length of a fracture trace formed as the intersection between a fracture and a 2-d sampling surface.

**Fracture trace length per unit area (L/A)** - mean summed trace length enclosed within a specified area, divided by that area.

**Sampling domain** - the region on which fracturing is to be measured or estimated. May be 3-d (e.g., the rock to be excavated for a tunnel), 2-d (e.g., rock pavements, pre-split road crops, tunnel walls) or 1-d (e.g., scanlines or small-diameter boreholes).
Terms currently used to describe amount of fracturing

Fracture spacing
Fracture density
Fracture intensity
Fracture porosity
Fracture frequency
Fracture persistence
Dimensionless fracture density
Degree of development

sampling domain
Fracture spacing  ambiguous; various interpretations for non-
 parallel or non-infinite fractures
Fracture density  various definitions, no consensus, often used in
 a qualitative sense
Fracture intensity  various definitions, no consensus, often used in
 a qualitative sense
Fracture porosity  well defined quantity; can be applied in 1, 2 or
 3-dimensions
Fracture frequency  OK for a specified sampling line. Direction-
 dependency must be noted explicitly
Fracture persistence  OK for intermittent or partially healed joints;
 otherwise ambiguous
Dimensionless fracture density  used in fracture mechanics;
 not useful for geologic fractures
Degree of development  OK, but qualitative
Fracture abundance:

umbrella term for amount of fracturing:

Scale-independent

Fracture abundance measures:

Density, Intensity, Porosity

defined in 1, 2 and 3-dimensions
Assumptions

- Fractures may be systematic or non-systematic, planar or non-planar, convex or non-convex.
- Fractures may possess cross-cutting, en echelon, termination, or any type of relationship with other fractures or structures.
- No spatial distributions are assumed.
- No particular orientation distributions are assumed, although certain special cases are examined.

We do assume that:

- Fracture distributions and locations are *independent* of the sampling domain.
- Fracture aperture is significantly smaller than diameter.
Sampling Problems

- Censoring
- Length-bias
- Truncation
- Orientation

Einstein and Baecher, RMRE 1983; Odling, JSG 1997; Mauldon RMRE 1998
Fracture Abundance Measures - Density

Fracture Density

*P_{10} (number of fractures / length of scanline) [L^{-1}]

*P_{20} (number of fractures / area of exposure)  [L^{-2}]

P_{30} (number of fractures / volume of rock mass) [L^{-3}]

* direction-dependent
Fracture Abundance Measures - Intensity

Fracture Intensity

*P_{10}(number of fractures / length of scanline) [L^{-1}]

*P_{21} (length of fracture traces / area of exposure) [L^{-1}]

P_{32} (area of fractures / volume of rock mass) [L^{-1}]

* direction-dependent
Fracture Abundance Measures - Porosity

Fracture Porosity

*P_{11} (thickness of fractures / length of scanline) [-]

*P_{22} (area of fracture traces / area of exposure) [-]

P_{33} (volume of fractures / volume of rock mass) [-]

* direction-dependent
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<td>P₁₀</td>
<td>P₁₁</td>
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<td>Area of Fractures per Unit Volume of Rock Mass (Volumetric Intensity)</td>
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<td>Volume of Fractures per Unit Volume of Rock Mass (Fracture Porosity)</td>
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</table>
Measuring Density (1)

\[ P_{10}(\theta) = \text{number of fractures} \ / \ \text{length of scanline} \]

Direct field measurements \( N/L \) along a straight line give unbiased estimates of \( P_{10}(\theta) \)

\[ P_{20}(\theta) = \text{number of fractures} \ / \ \text{area of exposure} \]

Direct field measurements \( N/A \) on a planar surface give unbiased estimates of \( P_{20}(\theta) \) as long as the number of fractures is determined correctly.

Question: What is the number of fractures for measurements of \( P_{20} \)? (and \( P_{30} \)?)
How many sailboats per unit area?
Measuring Density (3)

Not recommended: Identify subdomain containing uncensored sailboats. This violates assumed independence of structure and sampling domain; results are invalid.

Recommended: Identify and count unique associated points within sampling domain. This gives a scale-independent, unbiased estimate.

Sailboat density \( P_{20} = \frac{\# \text{ red flags}}{A} = \frac{\text{half total } \# \text{ of flags}}{A} = \frac{4}{A} \).
Note two-to-one association between trace ends and centers.

An unbiased estimate of trace density is therefore given by \( \frac{1}{2} \) number of trace ends divided by window area. For this example,

\[
\hat{P}_{20} = \frac{5/2}{36m^2} = 0.07m^{-2}
\]

Estimator is scale-independent

Mauldon, RMRE 1992
Measuring Intensity (1)

\[ P_{10} = \frac{\text{number of fractures}}{\text{length of scanline}} \]

SCANLINES: Direct field measurements \( N/L \) along a straight line give unbiased estimates of \( P_{10}(\theta) \)

\[ P_{21} = \frac{\text{length of fracture traces}}{\text{area of exposure}} \]

AREAS: Direct field measurements \( L'/A \) on a planar exposure give unbiased estimates of \( P_{21}(\theta) \)

\[ P_{32} = \frac{\text{area of fractures}}{\text{volume of rock mass}} \]

VOLUMES: Direct field measurements \( A'/V \) give unbiased estimates of \( P_{21}(\theta) \) --- In principle! - impossible to obtain directly

\( P_{32} \) is the “Holy Grail” of fracture abundance measures
Measuring Intensity (2)

\( P_{10} = \text{number of fractures/length of borehole} \)

**BOREHOLES/ ROCK CORE:** Direct field measurements \(N/L\) along a borehole give unbiased estimates of \(P_{10}(\theta)\)

\( P_{21} = \text{length of fracture traces / area of exposure} \)

**CIRCULAR SCANLINES:** Direct field measurements of \(n/4r\) on a circular scanline, where \(n\) is the number of trace intersections on the circle and \(r\) is the radius of the circle, give unbiased estimates of \(P_{21}(\theta)\) on a planar exposure.

Mauldon, Dunne & Rohrbaugh, JSG, in press
Measuring Porosity

Linear porosity

Areal porosity

Faults mapped directly from VSP vertical seismic profiling

Fractured oil reservoir - one-kilometer scale DFN model
Converting among Abundance Measures

--- A few examples ---

**Parallel fractures**

\[
P_{32} = \csc(\theta)P_{10}(\theta) \quad \text{(R. Terzaghi, 1965)}
\]

**Uniform distribution of fracture orientations**

\[
\begin{align*}
P_{32} &= 2 \, P_{10} \quad \text{from 1-d Measures} \\
P_{32} &= (4/\pi) \, P_{21} \quad \text{from 2-d Measures}
\end{align*}
\]

**Specified distribution of fracture orientation**

\[
\begin{align*}
P_{32} &= C_{31} \, P_{10} \quad \text{from 1-D Measures} \\
P_{32} &= C_{32} \, P_{21} \quad \text{from 2-D Measures}
\end{align*}
\]

**Conversion factors**

Some conversion factors are known analytically.
Conversion factors can be developed by simulation.

Dershowitz & Herda, USRM Symp. 1992;
Mauldon & Dershowitz (in preparation, IJRMMMS)
Conclusions

• The amount of fractures present in a rock mass can be described by fracture abundance measures: density, intensity and porosity.

• The abundance measures are scale-independent and are applicable in 1, 2 or 3-dimensions.

• 1-d and 2-d abundance measures are orientation-dependent.

• Estimates of these measures can be obtained from scanlines, scan-circles, borehole images, windows and tunnels.

• Together, the fracture abundance measures form a unified, self-consistent system.
Thank You

Amroth, South Wales, UK
Fracture Orientation Relative to Sampling Lines (Boreholes) and Planes (Tracemaps)

After Wang (2006)
Angle $\rho$ between fracture set mean pole and sampling line (borehole)

After Wang (2006)
Calculation of $C_{31}$ Conversion between $P_{10}$ and $P_{32}$ for Fisher Distributed Fractures

After Wang (2006)

$P_{32} = C_{31} \cdot P_{10}$

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$1/C_{31} = a \cos(b \rho) + c$

Coefficient

$\kappa$

After Wang (2006)
Calculation of $C_{32}$ Conversion between $P_{21}$ and $P_{32}$ for Fisher Distributed Fractures

After Wang (2006)

$P_{32} = C_{32} \cdot P_{21}$

Table 2.2. $1/C_{32}$ with different values of $\kappa$ and $\rho$.

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$1/C_{32} = \sin(b \rho - d \pi / 2) + c$